

## Type II pp-wave matrix models from point-like gravitons

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**ABSTRACT:** The BMN Matrix model can be regarded as a theory of coincident M-theory gravitons, which expand by Myers dielectric effect into the 2-sphere and 5-sphere giant graviton vacua of the theory. In this note we show that, in the same fashion, Matrix String theory in Type IIA pp-wave backgrounds arises from the action for coincident Type IIA gravitons. In Type IIB, we show that the action for coincident gravitons in the maximally supersymmetric pp-wave background gives rise to a Matrix model which supports fuzzy 3-sphere giant graviton vacua with the right behavior in the classical limit. We discuss the relation between our Matrix model and the Tiny Graviton Matrix theory of [1].

**KEYWORDS:** M(atrrix) Theories, p-branes, Penrose limit and pp-wave background.

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## 1. Introduction

Taking the point of view that the BFSS Matrix model [2] can be regarded as a theory of coincident M-theory gravitons, we showed in [3] that using the action for coincident gravitons proposed therein it is possible to go beyond the linear order approximation of [4]. This action was successfully used in [3, 5] for the study of giant gravitons [6, 7] in  $AdS_m \times S^n$  backgrounds, which are not linear perturbations to Minkowski. Moreover, in the M-theory maximally supersymmetric pp-wave background [8], this action, besides reproducing the BMN Matrix model [9], predicts a new quadrupolar coupling to the M-theory 6-form potential, which supports the so far elusive fuzzy 5-sphere giant graviton solution [10].

In this paper we will focus on Type II pp-wave Matrix models. These models share with the BMN Matrix model the removal of the flat directions and the existence of a large class of giant graviton supersymmetric vacua. We will mainly concentrate on two backgrounds: the Type IIA background that is obtained from the maximally supersymmetric pp-wave background of M-theory after dimensional reduction [11], and the maximally supersymmetric Type IIB pp-wave background [12]. We will simply refer to them as the Type IIA and Type IIB pp-wave backgrounds.

Several approaches have been taken in the literature for the study of Type II pp-wave Matrix models. Matrix String theory in the Type IIA pp-wave background has been studied in [13, 14]. The approach in [13] is to start from the supermembrane action in the maximally supersymmetric pp-wave background of M-theory, and then use the correspondence law

of [15] to reduce it to ten dimensions. Reference [14] constructs it, in turn, from the BMN Matrix action, using the 9-11 flip [16]. Matrix String models in more general pp-wave backgrounds have also been considered in [17], by studying certain deformations of ten dimensional  $N = 1$  SYM. These models include the BMN Matrix model when dimensionally reduced to one dimension, as well as the Matrix String theory of [13, 14], in two dimensions. In this reference a possible deformation of the IKKT Matrix String theory [18] which could be suitable for the study of Type IIB pp-wave backgrounds was also considered. General features about a Matrix String theory in the maximally supersymmetric pp-wave background of Type IIB were also discussed in [19]<sup>1</sup>. A Matrix String theory for this background was however not explicitly constructed till reference [1].

The approach taken in [1] is to regularize the light-cone 3-brane action in the Type IIB pp-wave background, in close analogy to the derivation in [21] of the BMN Matrix model from the light-cone supermembrane action. The light-cone 3-brane carries  $N$  units of light-cone momentum, and some of its vacua are finite size 3-branes with zero light-cone energy, i.e. giant gravitons<sup>2</sup>. In close analogy to the description in [9, 1] proposes a description of the 3-sphere vacua in terms of  $N$  expanding gravitons, each carrying one unit of light-cone momentum, the so-called *tiny* gravitons. The resulting Matrix model, a one dimensional  $U(N)$  gauge theory, is referred as the Tiny Graviton Matrix theory.

Keeping in mind that the BMN Matrix model can be regarded as a theory of coincident M-theory gravitons [10], which expand by Myers dielectric effect into the 2-sphere and 5-sphere giant graviton vacua of the theory, one would expect that the Tiny Graviton Matrix theory of [1] could, in the same fashion, be regarded as a theory of Type IIB coincident gravitons, which expand by dielectric effect into the 3-sphere vacua. The tiny gravitons of [1] would then simply be coincident Type IIB point-like gravitons. In fact, it was shown in [23, 24] that the giant graviton solutions of the  $AdS_5 \times S^5$  and  $AdS_3 \times S^3 \times T^4$  Type IIB backgrounds can be described microscopically using the action for coincident gravitons constructed in [23]. This action contains the right multipole moment couplings to explain the expansion of the gravitons into fuzzy 3-spheres or fuzzy cylinders, respectively. Moreover, this action is a  $U(N)$  gauge theory, in which the non-Abelian vector field is associated to (wrapped) D3-branes “ending” on the gravitons<sup>3</sup>. In this construction each graviton carries one unit of light-cone momentum, and the enhancement from the  $U(1)$  gauge theory, for a single graviton, to a  $U(N)$  gauge theory, for  $N$  gravitons, takes place identically than in a system of coincident D-branes. This is in agreement with the discussion in [1], and with the results in [22].

In this article we pursue further the line of research initiated in [10], and show that the Matrix models that have been constructed in the literature in Type II pp-wave backgrounds can be obtained from the actions for Type II coincident gravitons constructed in [3, 25, 23].

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<sup>1</sup>See also [20].

<sup>2</sup>Fixing the light-cone gauge corresponds to going to the rest frame of the giant graviton. There is another solution consisting on a zero-size 3-brane with the same energy, i.e. a point-like graviton.,SS

<sup>3</sup>The D3-branes are wrapped on two isometric directions of the action, which in the pp-wave background are the light-cone direction and a combination of the two  $S^1$  fibres in the decomposition of the 3-spheres of the background as  $U(1)$  bundles over  $S^2$ .

Matrix String theory in the Type IIA pp-wave background [13, 14] is reproduced exactly, whereas in Type IIB we obtain a new Matrix model which supports fuzzy 3-sphere giant graviton solutions with the right behavior in the large  $N$  limit. We discuss with some detail the relation between this Matrix model and the Tiny Graviton Matrix theory of [1] throughout the paper.

The article is organized as follows. section 2 is devoted to the study of the type IIA pp-wave Matrix model. After briefly reviewing the background in subsection 2.1 we present the action describing Type IIA coincident gravitons in subsection 2.2. This action is a completion of the truncated action derived in [3, 25], which, as we discuss, is not suitable for the study of this background. We particularize the action constructed in subsection 2.2 to the Type IIA pp-wave background in subsection 2.3 and show the perfect agreement with the matrix actions of [13, 14]. Finally in subsection 2.4 we discuss some of the fuzzy sphere vacuum solutions. section 3 is devoted to the study of the Type IIB pp-wave Matrix model. We start in subsection 3.1 by rewriting the background in coordinates adapted to our construction. In subsection 3.2 we recall the action for type IIB coincident gravitons of [23]. We see that this action is adequate for the study of the pp-wave background. In subsection 3.3 we present our proposal for the Matrix model. Finally we summarize in subsection 3.4 some of the fuzzy 3-sphere vacua of the model. We end in section 4 with some conclusions.

## 2. The Type IIA Matrix Model

### 2.1 The background

We consider the Type IIA pp-wave background:

$$\begin{aligned}
 ds^2 &= -2dx^+ dx^- - \beta(dx^+)^2 + dx_1^2 + \dots + dx_8^2, \\
 C_+^{(1)} &= -\frac{1}{3}\mu x^4, \quad C_{+ij}^{(3)} = -\frac{\mu}{3}\epsilon_{ijk}x^k; \quad i, j, k = 1, 2, 3
 \end{aligned}
 \tag{2.1}$$

which is obtained by reducing along an isometric  $SO(6)$  direction the maximally supersymmetric pp-wave background of M-theory [11]. This background preserves 24 of the original 32 supersymmetries [22, 26]. Here

$$\beta = \left(\frac{\mu}{3}\right)^2(x_1^2 + \dots + x_4^2) + \left(\frac{\mu}{6}\right)^2(x_5^2 + \dots + x_8^2).
 \tag{2.2}$$

We are interested in constructing a Matrix model describing the dynamics of gravitational waves in this background, in the sector with momentum  $p^+ = -p_- = N/R$ .

### 2.2 The action for Type IIA gravitons

An action describing coincident gravitational waves in Type IIA backgrounds has been constructed to linear order in the background fields in [25]. This action is, however, not suitable for the study of the Type IIA background (2.1), because  $(C_+^{(1)})^2$  contributes with a quadratic power of the  $x^4$  transverse scalar and therefore gives a contribution to the leading order expansion of the action (see next subsection). In this section we construct

an action for Type IIA waves that goes beyond the linear approximation, and is suitable for the study of the Type IIA pp-wave background (2.1) to the desired order.

Our starting point is the action for coincident M-theory gravitational waves constructed in [3]. This action goes beyond the linear order approximation, and has the same regime of validity of Myers action for coincident D-branes [27]<sup>4</sup>:

$$S = - \int d\tau \text{STr} \left\{ k^{-1} \sqrt{- \left[ g_{\mu\nu} \mathcal{D}_\tau X^\mu \mathcal{D}_\tau X^\nu + E_{\tau i} (Q^{-1} - \delta)_k^i E^{kj} E_{j\tau} \right] \det Q} \right\} + \int d\tau \text{STr} \left\{ P \left[ k^{-2} k^{(1)} \right] - iP \left[ (i_X i_X) C^{(3)} \right] + \frac{1}{2} P \left[ (i_X i_X)^2 i_k C^{(6)} \right] + \dots \right\} \quad (2.3)$$

where

$$E_{\mu\nu} = g_{\mu\nu} + k^{-1} (i_k C^{(3)})_{\mu\nu}, \quad Q_j^i = \delta_j^i + ik [X^i, X^k] E_{kj}, \quad i = 1, \dots, 9. \quad (2.4)$$

Here we have taken units in which the tension of a single graviton is equal to one. In this action the direction of propagation of the waves appears as a special isometric direction, with Killing vector  $k^\mu$ .  $k^2$  and  $k^{(1)}$  are defined as  $k^2 = g_{\mu\nu} k^\mu k^\nu$  and  $k_\mu^{(1)} = g_{\mu\nu} k^\nu$ .  $k^{-2} k^{(1)}$  is then the momentum operator along the isometric direction. Consistently with this isometry the pull-backs into the worldvolume are taken with gauge covariant derivatives [28]

$$\mathcal{D}_\tau X^\mu = \partial_\tau X^\mu - k^{-2} k_\nu \partial_\tau X^\nu k^\mu. \quad (2.5)$$

In this way the dependence on the isometric direction is effectively eliminated from the action. This action is in fact a gauge fixed action in which the  $U(N)$  vector field, associated to M2-branes (wrapped on the direction of propagation) ending on the waves, has been taken to vanish,  $A_\tau = 0$ . In this gauge  $U(N)$  covariant derivatives reduce to ordinary derivatives, and gauge covariant derivatives can be defined using ordinary derivatives as in (2.5).

We can obtain the action describing coincident Type IIA gravitons by reducing the action (2.3) along a transverse direction. Applying the dimensional reduction rules to the eleven dimensional metric it is clear that quadratic powers of the RR 1-form potential show up. Given that in the pp-wave background  $C_+^{(1)} = -\mu x^4/3$ , these couplings have to be included in order to construct a Matrix model which contains quadratic powers of the transverse scalars. For simplicity we restrict ourselves to backgrounds in which the NS-NS 2-form potential vanishes, all components but the time component and the component on the direction of propagation of the RR 1-form potential are zero, and  $k_i = 0$ . This is indeed a suitable truncation for the pp-wave background (2.1).

We obtain for the BI action

$$S^{\text{BI}} = - \int d\tau \text{STr} \left\{ \frac{1}{\sqrt{k^2 + e^{2\phi} (i_k C^{(1)})^2}} \sqrt{\left( \mathbb{1} - (k^2 + e^{2\phi} (i_k C^{(1)})^2) [A, X]^2 \right) \det Q} \cdot \sqrt{- \left[ g_{\mu\nu} \mathcal{D}_\tau X^\mu \mathcal{D}_\tau X^\nu + \frac{k^2 e^{2\phi}}{k^2 + e^{2\phi} (i_k C^{(1)})^2} \left( C_\mu^{(1)} \mathcal{D}_\tau X^\mu + F \right)^2 \right]} \right\} \quad (2.6)$$

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<sup>4</sup>We refer the reader to reference [3] for more details.

Here

$$\begin{aligned}
 E_{\mu\nu} &= g_{\mu\nu} + \frac{e^\phi}{\sqrt{k^2 + e^{2\phi}(\mathbf{i}_k C^{(1)})^2}} (\mathbf{i}_k C^{(3)})_{\mu\nu} \\
 Q_j^i &= \delta_j^i + i[X^i, X^k] e^{-\phi} \sqrt{k^2 + e^{2\phi}(\mathbf{i}_k C^{(1)})^2} E_{kj}
 \end{aligned}
 \tag{2.7}$$

and  $i$  now runs from 1 to 8.  $A$  is the scalar field that comes from the reduction of the eleventh transverse direction and  $F$  is its field strength.  $F$  forms an invariant field strength with the pull-back of the RR 1-form potential, and therefore  $A$  is associated to D0-branes ending on the waves. The first square root in (2.6) comes from the reduction of the determinant of the nine dimensional  $Q$  matrix, whereas the second square root comes from the reduction of the pull-back of the metric. We should mention that in this action we have made a further truncation. We have omitted those terms coming from the reduction of  $E_{\tau i}(Q^{-1} - \delta)_k^i E^{kj} E_{j\tau}$ . This is justified because these terms contribute to higher order on the transverse scalars.

Dimensionally reducing the CS action we get:

$$\begin{aligned}
 S^{\text{CS}} &= \int d\tau \text{STr} \left\{ P \left[ k^{-2} k^{(1)} \right] + \frac{e^{2\phi} \mathbf{i}_k C^{(1)}}{k^2 + e^{2\phi}(\mathbf{i}_k C^{(1)})^2} \left( P \left[ C^{(1)} \right] + F \right) - iP \left[ (\mathbf{i}_X \mathbf{i}_X) C^{(3)} \right] \right. \\
 &\quad \left. + \frac{1}{2} P \left[ (\mathbf{i}_X \mathbf{i}_X)^2 \mathbf{i}_k B^{(6)} \right] + \dots \right\}
 \end{aligned}
 \tag{2.8}$$

From here we see that the Type IIA gravitons propagate along the same isometric direction, consistently with the fact that the isometry is inherited when we reduce along a transverse direction. We also find dielectric couplings to the RR 3-form and the NS-NS 6-form potentials, which would be responsible for the expansion of the gravitons into D2-branes and NS5-branes in suitable backgrounds. We have omitted couplings to higher order background potentials and products of different background fields contracted with the non-Abelian scalars because they will not play a role in the pp-wave background that we consider in this paper.

### 2.3 The Matrix model

We can now particularize the actions (2.6) and (2.8) to the Type IIA pp-wave background (2.1). We are interested in describing waves with non-vanishing light-cone momentum  $p_-$ . However the actions (2.6) and (2.8) are singular for the choice  $k^\mu = \delta_-^\mu$ , since  $k^2 = g_{--}$  vanishes in the pp-wave background. A natural way to regularize the action is to undo the Penrose limit, keep the waves propagating in the  $\psi$ -direction of the original eleven dimensional AdS background, and finally take the Penrose limit  $L \rightarrow \infty$ . Since, in the notation of (2.1), the light-cone coordinates are related to the AdS time and  $\psi$  coordinate through

$$x^+ = \frac{3}{2\mu}(t + \psi), \quad x^- = \frac{\mu L^2}{3}(t - \psi),
 \tag{2.9}$$

$p_-$  and  $p_\psi$  are related through

$$p_\psi = -p_- \frac{\mu L^2}{3}.
 \tag{2.10}$$

If we take the gravitons propagating along the  $\psi$  direction with momentum  $p_\psi = N$  we have that  $p_- = -N/R$  if  $R$  is related to the AdS radius and the mass scale as  $R = \mu L^2/3$ .

Therefore, we take  $k^\mu = \delta_\psi^\mu$  in the actions (2.6) and (2.8). Moreover, we take light-cone gauge and identify  $x^+$  with the worldline time. Expanding the action (2.6) to leading order (in  $\lambda = 2\pi\alpha'$ ) we find<sup>5</sup>

$$S^{\text{BI}} = -\frac{\mu}{3} \int dx^+ \text{STr} \left\{ \mathbb{1} - \frac{L^2}{4} [X, X]^2 - \frac{L^2}{2} [A, X]^2 + \frac{i}{2} \epsilon_{ijk} X^i X^j X^k + \frac{9\tilde{\beta}^2}{4\mu^2 L^2} - \frac{9}{2\mu^2 L^2} \dot{X}^2 - \frac{9}{2\mu^2 L^2} F \left( F - \frac{\mu}{3} X^4 \right) \right\} \quad (2.11)$$

and

$$S^{\text{CS}} = \int dx^+ \text{STr} \left\{ \frac{\mu}{3} \left( 1 - \frac{9\tilde{\beta}}{4\mu^2 L^2} \right) - \frac{1}{2L^2} X^4 F - i \frac{\mu}{6} \epsilon_{ijk} X^i X^j X^k \right\} \quad (2.12)$$

where

$$\tilde{\beta} = \beta - \frac{\mu^2}{9} (X^4)^2, \quad (2.13)$$

$i = 1, 2, 3$  and we denote the non-Abelian transverse scalars with capital letters.

Therefore, the final action reads

$$S = \int dx^+ \text{STr} \left( \frac{1}{2R} \dot{X}^2 - \frac{\tilde{\beta}}{2R} + \frac{R}{4} [X, X]^2 + \frac{R}{2} [A, X]^2 + \frac{1}{2R} F \left( F - \frac{2\mu}{3} X^4 \right) - i \frac{\mu}{3} \epsilon_{ijk} X^i X^j X^k \right) \quad (2.14)$$

where we have substituted  $R = \mu L^2/3$ . This action is in perfect agreement with the results in [13] and [14] when one makes the truncation  $\partial_\sigma X^\mu = 0$  (see the discussion below).

Indeed, it was shown in [29, 25] that Matrix String theory has an alternative interpretation as describing the dynamics of coincident Type IIA gravitons. The idea in [29, 25] is that since Matrix String theory describes string states with fixed light cone momentum, it could, in some limit, effectively describe gravitons. Explicitly, Matrix String theory is constructed by compactifying M-theory on the 9th direction, and then performing the 9-11 flip [16]. However, when one considers weakly curved backgrounds the 9th direction appears as a special isometric direction on which neither the background fields nor the currents depend. This is translated into a reduced,  $SO(8)$  transverse rotationally invariant action. One can however rewrite the action in terms of ten dimensional pull-backs into a one dimensional worldvolume by using the techniques of gauged sigma models.

The 9th direction is interpreted as the spatial worldsheet direction of the string,  $\sigma$ . If one makes the truncation  $\partial_\sigma X^\mu = 0$  and let  $k^\mu$  be the Killing vector pointing along the 9th direction one can achieve invariance under the local isometric transformations generated by  $k^\mu$  by introducing gauge covariant derivatives as in (2.5). Using gauge covariant pull-backs, constructed with these gauge covariant derivatives, it is possible to eliminate the pull-back of the isometric coordinate, and to reproduce the isometric couplings in the Matrix String action in a manifestly covariant way (see [25]).

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<sup>5</sup>Note that in our units  $\lambda = 1$ . This approximation is however the usual one taken in non-Abelian BI actions (see [27]), based on the fact that these actions are good to describe the system of branes when they are distances away less than the Planck length.

Consistently with this discussion we reproduce, using the action for Type IIA gravitons, the subsector of the Matrix String theory constructed in [13, 14] satisfying  $\partial_\sigma X^\mu = 0$ .

## 2.4 The fuzzy sphere solutions

The Matrix model (2.14) admits fuzzy 2-sphere giant graviton solutions. One is a static fuzzy sphere located at  $x^4 = \dots = x^8 = 0$ , identical to the 2-sphere solution of the BMN Matrix model but from the fact that it preserves just eight supersymmetries due to the toroidal compactification from M-theory [13]. A more general solution is considered in [14] in which the static fuzzy sphere is located in an arbitrary point in the  $x^4$  direction, which preserves as well eight supersymmetries. In this section we are going to show however that the  $x^4$  location and the radius of the fuzzy 2-sphere must satisfy the relation

$$x^4 r = -\frac{m\sqrt{N^2 - 1}}{2N}, \tag{2.15}$$

where  $m$  is an integer, in agreement with the results found in [30] in the large  $N$  limit.

Let us first discuss the solutions in [30]. In this reference spherical D2-brane giant graviton solutions are studied using a test D2-brane in the pp-wave background, carrying light-cone momentum. It is found that 2-sphere solutions with non-vanishing  $x^4$  are possible when a magnetic field inducing D0-brane charge in the configuration is switched on. Then the radius of the spherical D2-brane and its  $x^4$  location must satisfy the relation

$$x^4 r = -m/2, \tag{2.16}$$

where  $m$  is the D0-brane charge induced in the worldvolume. This result is supported by a microscopical calculation in terms of non-Abelian  $m$  D0-branes expanding into the fuzzy 2-sphere by dielectric effect.

We now show that a similar condition for the radius of the spherical fuzzy D2-brane and its  $x^4$  location is predicted within the Matrix model description.

Let us start by taking in (2.14) the fuzzy 2-sphere ansatz:

$$X^i = \frac{r}{\sqrt{C_N}} J^i, \quad i = 1, 2, 3 \tag{2.17}$$

with  $J^i$  the generators of  $SU(2)$  in an  $N$  dimensional representation (in our notation  $[J^i, J^j] = 2i\epsilon^{ijk} J^k$ ) and  $C_N$  the quadratic Casimir in this representation. Take as well  $X^4 = \text{constant}$  and Abelian (we will denote it as  $x^4$ ),  $X^5 = \dots = X^8 = 0$ , and  $F$  Abelian. We then obtain an action

$$S = -\frac{N}{2R} \int dx^+ \left[ \left( \frac{\mu}{3} - \frac{2Rr}{\sqrt{N^2 - 1}} \right) r^2 - F \left( F - \frac{2}{3} \mu x^4 \right) \right], \tag{2.18}$$

and, Legendre transforming  $F$ , a Hamiltonian

$$H = \frac{N}{2R} \left[ \left( \frac{\mu}{3} - \frac{2Rr}{\sqrt{N^2 - 1}} \right)^2 r^2 + \left( \frac{\mu x^4}{3} + \frac{p_A R}{N} \right)^2 \right], \tag{2.19}$$

where  $p_A$  is the conjugate momentum of the scalar field  $A$ . Recalling that  $A$  has its origin on the eleventh transverse scalar,  $p_A$  is the momentum along the eleventh direction, which is interpreted in ten dimensions as D0-brane charge.



Now, minimizing with respect to  $r$  and  $x^4$ , we find two zero energy solutions, one with zero radius, the point-like graviton, and a second one with

$$r = \frac{\mu\sqrt{N^2 - 1}}{6R}, \tag{2.20}$$

which corresponds to the giant graviton. Both solutions are located at

$$x^4 = -\frac{3mR}{\mu N}, \tag{2.21}$$

for  $m$  D0-brane charge.

Therefore, microscopically, the radius and the position in  $x^4$  of the giant graviton must be related through

$$x^4 r = -\frac{m\sqrt{N^2 - 1}}{2N}. \tag{2.22}$$

This reproduces the relation (2.16) of [30] in the large  $N$  limit. Moreover, the Hamiltonian (2.19), derived using the Matrix model, coincides in the large  $N$  limit with the Hamiltonian describing the spherical D2-brane with momentum  $N$  and D0-brane charge  $m$  of [30], as we now show.

A classical spherical D2-brane in the pp-wave background (2.1) with  $x^- = x^-(x^+)$  sitting at a constant  $x^4$  at  $x^5 = \dots = x^8 = 0$ , and carrying D0-brane charge  $m$ , is described by a Lagrangian

$$S = -4\pi T_2 \int dx^+ \left\{ \sqrt{\frac{\mu^2}{9} (r^2 + (x^4)^2)} + 2i^- \sqrt{r^4 + \frac{m^2}{4}} - \frac{\mu}{3} \left( r^3 - \frac{m}{2} x^4 \right) \right\}, \tag{2.23}$$

where we have substituted

$$F_{\theta\phi} = \frac{m}{2} \sin \theta, \tag{2.24}$$

which is the magnetic field inducing  $m$  D0-brane charge in the worldvolume.

Legendre transforming with respect to  $\dot{x}^-$  we arrive at the following Hamiltonian in terms of the canonically conjugated momentum  $p_-$

$$H = -\frac{p_-}{2} \left[ \left( \frac{\mu}{3} + \frac{4\pi T_2 r}{p_-} \right)^2 r^2 + \left( \frac{\mu x^4}{3} - \frac{2\pi T_2 m}{p_-} \right)^2 \right]. \tag{2.25}$$

This expression coincides exactly in the large  $N$  limit with the microscopical Hamiltonian (2.19), when we take into account that in our units  $2\pi T_2 = 1$ .

The minimum energy,  $E = 0$ , is reached when:

$$r = -\frac{\mu p_-}{12\pi T_2}, \quad x^4 = \frac{6\pi T_2 m}{\mu p_-}, \tag{2.26}$$

which also agree exactly in the large  $N$  limits with expressions (2.20) and (2.21).

We can conclude that the classical 2-sphere solution found in [30] is correctly reproduced within the Matrix model description. Since this 2-sphere giant graviton solution carries both momentum charge (in our notation,  $N$ ), and D0-brane charge (in our notation,  $m$ ) it is possible to describe it microscopically either as  $N$  gravitational waves with

D0-brane charge  $m$ , expanding due to their (dielectric) coupling to the RR 3-form potential, or as  $m$  D0-branes moving along the  $x^-$  direction with momentum  $N$  (expanding as well due to their coupling to the 3-form potential). This explains why the microscopical description of [30] in terms of D0-branes agrees with the Matrix model description. The Matrix model provides however a unified set-up for the microscopical study of giant graviton configurations in both Type II and M theories.

### 3. The Type IIB Matrix Model

In this section we propose a Matrix model for the maximally supersymmetric pp-wave background of Type IIB. This Matrix model is constructed from the action describing coincident Type IIB gravitons.

#### 3.1 The background

We start by recalling the form of the maximally supersymmetric pp-wave background of Type IIB [12]. It arises as the Penrose limit of the  $AdS_5 \times S^5$  background

$$\begin{aligned}
 ds^2 &= L^2(-\cosh^2 \rho d\tau^2 + d\rho^2 + \sinh^2 \rho d\Omega_3^2 + d\theta^2 + \cos^2 \theta d\psi^2 + \sin^2 \theta d\tilde{\Omega}_3^2) \\
 C_{\tau\alpha_1\alpha_2\alpha_3}^{(4)} &= -L^4 \sinh^4 \rho \sqrt{g_\alpha}, \quad C_{\psi\gamma_1\gamma_2\gamma_3}^{(4)} = -L^4 \sin^4 \theta \sqrt{g_\gamma}
 \end{aligned}
 \tag{3.1}$$

where  $\{\alpha_i\}$  and  $\{\gamma_i\}$  are, respectively, the angles parametrizing the 3-spheres contained in the AdS and  $S$  parts of the geometry:

$$\begin{aligned}
 d\Omega_3^2 &= d\alpha_1^2 + \sin^2 \alpha_1 (d\alpha_2^2 + \sin^2 \alpha_2 d\alpha_3^2), \\
 d\tilde{\Omega}_3^2 &= d\gamma_1^2 + \sin^2 \gamma_1 (d\gamma_2^2 + \sin^2 \gamma_2 d\gamma_3^2),
 \end{aligned}
 \tag{3.2}$$

and  $\sqrt{g}$  is the volume element on the unit 3-sphere.

Defining

$$x^+ = \frac{1}{2\mu}(\tau + \psi), \quad x^- = \mu L^2(\tau - \psi), \quad \rho = \frac{r}{L}, \quad \theta = \frac{y}{L}
 \tag{3.3}$$

and taking  $L \rightarrow \infty$  one gets [31]

$$\begin{aligned}
 ds^2 &= -2dx^+ dx^- - \mu^2(r^2 + y^2)(dx^+)^2 + dr^2 + r^2 d\Omega_3^2 + dy^2 + y^2 d\tilde{\Omega}_3^2 \\
 C_{+\alpha_1\alpha_2\alpha_3}^{(4)} &= -\mu r^4 \sqrt{g_\alpha}, \quad C_{+\gamma_1\gamma_2\gamma_3}^{(4)} = -\mu y^4 \sqrt{g_\gamma}.
 \end{aligned}
 \tag{3.4}$$

We are interested in constructing a Matrix model describing the dynamics of gravitons in this background in the sector with momentum  $p^+ = -p_- = N/R$ . In order to do this it is convenient to describe the 3-spheres in (3.4) as Hopf-fiberings,  $p : S^3 \rightarrow S^2$ , using the round metric for  $S^3$ :

$$d\Omega_3^2 = \frac{1}{4} \left( (d\chi - A)^2 + d\Omega_2^2 \right)
 \tag{3.5}$$

where, in Euler angles:

$$A = -\cos \chi_1 d\chi_2, \quad d\Omega_2^2 = d\chi_1^2 + \sin^2 \chi_1 d\chi_2^2.
 \tag{3.6}$$

Using now Cartesian coordinates to describe the 2-spheres we can write the background metric and potentials (3.4) as

$$\begin{aligned}
 ds^2 &= -2dx^+ dx^- - \mu^2(r^2 + y^2)(dx^+)^2 + dr^2 + \frac{r^2}{4} \left( (d\chi - A)^2 + dx_1^2 + dx_2^2 + dx_3^2 \right) + \\
 &+ dy^2 + \frac{y^2}{4} \left( (d\tilde{\chi} - \tilde{A})^2 + dz_1^2 + dz_2^2 + dz_3^2 \right), \\
 C_{\chi+ij}^{(4)} &= \frac{1}{8} \mu r^4 \epsilon_{ijk} x^k, \quad i, j = 1, 2, 3, \\
 C_{\tilde{\chi}+ab}^{(4)} &= \frac{1}{8} \mu y^4 \epsilon_{abc} z^c, \quad a, b = 1, 2, 3
 \end{aligned} \tag{3.7}$$

where  $\vec{x}$  and  $\vec{z}$  parametrize points in  $\mathbb{R}^3$ . In these coordinates:

$$\begin{aligned}
 A &= -\frac{x_3}{x_1^2 + x_2^2} (x_1 dx_2 - x_2 dx_1) \\
 \tilde{A} &= -\frac{z_3}{z_1^2 + z_2^2} (z_1 dz_2 - z_2 dz_1).
 \end{aligned} \tag{3.8}$$

Note that in these coordinates we have reduced the explicit invariance of the background from  $U(1)^2 \times (SO(4))^2$  to  $U(1)^2 \times (SO(3) \times U(1))^2$ , though the whole invariance should still be present in a non-manifest way.

### 3.2 The action for Type IIB gravitons

The action describing coincident Type IIB gravitons was constructed in [23]. Like the action for Type IIA gravitons, it goes beyond the linear order approximation and has the same regime of validity than Myers action for coincident D-branes. It is given by:

$$\begin{aligned}
 S &= - \int d\tau \text{STr} \left\{ k^{-1} \sqrt{- \left[ g_{\mu\nu} \mathcal{D}_\tau X^\mu \mathcal{D}_\tau X^\nu + E_{\tau i} (Q^{-1} - \delta)_k^i E^{kj} E_{j\tau} \right] \det Q} \right\} + \\
 &+ \int d\tau \text{STr} \left\{ P \left[ k^{-2} k^{(1)} \right] - iP \left[ (i_X i_X) i_l C^{(4)} \right] + \dots \right\}
 \end{aligned} \tag{3.9}$$

where

$$E_{\mu\nu} = g_{\mu\nu} - k^{-1} l^{-1} e^\phi (i_k i_l C^{(4)})_{\mu\nu}, \quad Q_j^i = \delta_j^i + i[X^i, X^k] e^{-\phi} k l E_{kj}, \quad i = 1, \dots, 7 \tag{3.10}$$

and the gauge covariant derivatives are defined as

$$\mathcal{D}_\tau X^\mu = \partial_\tau X^\mu - k^{-2} k_\nu \partial_\tau X^\nu k^\mu - l^{-2} l_\nu \partial_\tau X^\nu l^\mu. \tag{3.11}$$

A detailed discussion of this action can be found in [23]. Like the action for Type IIA gravitons, the direction of propagation appears as a special isometric direction, with Killing vector  $k^\mu$ . In this case however there is a second isometric direction, with Killing vector  $l^\mu$ , which is inherited from the T-duality transformation involved in the construction. Although in the Abelian limit the dependence in this direction can be restored, this does not happen in the non-Abelian case (see the discussion in [23]). Note that it is precisely due to the existence of this second isometry that the RR 4-form potential can couple in

the action (3.9). Otherwise the dielectric couplings in (3.9) and in (3.10) would not be possible<sup>6</sup>. Therefore, the action (3.9) is suitable for the study of gravitons which propagate in backgrounds with isometric directions. This is indeed the case in the pp-wave background, where there are two isometric directions associated to the two fibres in the Hopf decomposition of the transverse 3-spheres.

The action (3.9) is, again, a gauge fixed action, in which the  $U(N)$  vector field, associated in this case to D3-branes wrapped on the two isometric directions, is set to zero. In this gauge  $U(N)$  covariant derivatives reduce to ordinary derivatives, as in (3.11).

### 3.3 The Matrix model

We can now particularize the action (3.9) to the background (3.7). As in subsection 2.3 we take light-cone gauge and identify  $x^+$  with the worldline time. We take as well the gravitons propagating in the  $\psi$  direction, i.e.  $k^\mu = \delta_\psi^\mu$ . Again, this is equivalent to taking the gravitons with momentum  $p_-$ , since  $p_\psi$  and  $p_-$  are related through the change of coordinates (3.3) as

$$p_\psi = -\mu L^2 p_- . \tag{3.12}$$

Therefore we describe the sector of the theory with light-cone momentum  $p_- = -N/R$  taking  $p_\psi = N$  and  $R = \mu L^2$ . Doing this we avoid the singularities that arise in the action if we simply take  $k^\mu = \delta_-^\mu$ , due to the fact that  $k^2 = g_{--} = 0$ .

In order to identify the second isometric direction we change coordinates

$$\xi = \frac{\chi + \tilde{\chi}}{2}, \quad \tilde{\xi} = \frac{\chi - \tilde{\chi}}{2} \tag{3.13}$$

and take  $l^\mu = \delta_\xi^\mu$ . This choice preserves the  $\mathbb{Z}_2$  symmetry  $\chi \leftrightarrow \tilde{\chi}$ ,  $r \leftrightarrow y$ ,  $x \leftrightarrow z$  of the background. Then we have

$$C_{\xi+ij}^{(4)} = \frac{1}{8} \mu r^4 \epsilon_{ijk} x^k, \quad C_{\xi+ab}^{(4)} = \frac{1}{8} \mu y^4 \epsilon_{abc} z^c . \tag{3.14}$$

These two potentials couple in both the CS and BI parts of the action. This will allow the existence of zero energy solutions corresponding to expansions of the gravitons into the two 3-spheres contained in the geometry.

We make the ansatz that the radii of the two 3-spheres are commutative, consistently with the symmetries of the background, and we restrict the use of capital letters for the non-commutative scalars. We find for the CS action:

$$S^{\text{CS}} = \mu \int dx^+ \text{STr} \left\{ \mathbb{1} - \frac{1}{4L^2} (r^2 + y^2) - i \frac{r^4}{16} \epsilon_{ijk} X^i X^j X^k - i \frac{y^4}{16} \epsilon_{abc} Z^a Z^b Z^c \right\}, \tag{3.15}$$

and for the BI action

$$S^{\text{BI}} = -\mu \int dx^+ \text{STr} \left\{ \mathbb{1} + \frac{r^2 + y^2}{4L^2} - \frac{1}{2\mu^2 L^2} \left( \dot{r}^2 + \dot{y}^2 + \frac{r^2}{4} \dot{X}^2 + \frac{y^2}{4} \dot{Z}^2 \right) + \right.$$

---

<sup>6</sup>One could expect in principle a coupling of the form  $(i_X i_X) i_k C^{(4)}$  in the CS action. However, such a coupling does not arise in the T-duality transformation involved in the construction of (3.9), and, moreover, it vanishes in the  $AdS_5 \times S^5$  background. Indeed, in this background, the coupling that is responsible for the existence of the dual giant graviton solution is  $(i_X i_X) i_l C^{(4)}$  [23].

$$\begin{aligned}
& -\frac{1}{2\mu^2 L^2} \left( \frac{r^2 y^2}{r^2 + y^2} \right) \left( \dot{\xi} - \frac{A_i \dot{X}^i - \tilde{A}_a \dot{Z}^a}{2} \right) \left( \dot{\xi} - \frac{A_j \dot{X}^j - \tilde{A}_b \dot{Z}^b}{2} \right) + \\
& -\frac{1}{256} L^2 (r^2 + y^2) \left( r^4 [X, X]^2 + y^4 [Z, Z]^2 + 2r^2 y^2 [X, Z]^2 \right) + \\
& + i \frac{r^4}{16} \epsilon_{ijk} X^i X^j X^k + i \frac{y^4}{16} \epsilon_{abc} Z^a Z^b Z^c \} \tag{3.16}
\end{aligned}$$

where we have expanded the action (3.9) to leading order and  $A \equiv A_i dx^i$ ,  $\tilde{A} \equiv \tilde{A}_a dz^a$ .

Notice that  $\xi$  only appears in the quadratic term in the second line of the action. Integrating it out through its equation of motion we finally get

$$\begin{aligned}
S^{\text{BI}} = & -\mu \int dx^+ \text{STr} \left\{ \mathbb{1} - \frac{1}{2\mu^2 L^2} \left( \dot{r}^2 + \dot{y}^2 + \frac{r^2}{4} \dot{X}^2 + \frac{y^2}{4} \dot{Z}^2 \right) + \frac{r^2 + y^2}{4L^2} + \right. \\
& -\frac{1}{256} L^2 (r^2 + y^2) \left( r^4 [X, X]^2 + y^4 [Z, Z]^2 + 2r^2 y^2 [X, Z]^2 \right) + \\
& \left. + i \frac{r^4}{16} \epsilon_{ijk} X^i X^j X^k + i \frac{y^4}{16} \epsilon_{abc} Z^a Z^b Z^c \right\} \tag{3.17}
\end{aligned}$$

Combining (3.17) and (3.15) and taking into account that  $R = \mu L^2$  we can finally read our proposal for the Type IIB Matrix model:

$$\begin{aligned}
S = & \int dx^+ \text{STr} \left\{ \frac{1}{2R} \left( \dot{r}^2 + \dot{y}^2 + \frac{r^2}{4} \dot{X}^2 + \frac{y^2}{4} \dot{Z}^2 \right) - \frac{\mu^2}{2R} (r^2 + y^2) + \right. \\
& + \frac{1}{256} R (r^2 + y^2) \left( r^4 [X, X]^2 + y^4 [Z, Z]^2 + 2r^2 y^2 [X, Z]^2 \right) + \\
& \left. - i \frac{\mu}{8} r^4 \epsilon_{ijk} X^i X^j X^k - i \frac{\mu}{8} y^4 \epsilon_{abc} Z^a Z^b Z^c \right\} \tag{3.18}
\end{aligned}$$

Notice that this action is symmetric under the interchange  $r \leftrightarrow y$ ,  $X \leftrightarrow Z$ , like the background.

The Type IIB Matrix theory given by (3.18) is a  $U(N)$  gauge theory built up with six non-Abelian scalars,  $X^i$ ,  $i = 1, 2, 3$ , and  $Z^a$ ,  $a = 1, 2, 3$  plus two Abelian ones,  $r$  and  $y$ . The gauge field is set to zero through the gauge fixing condition  $A_\tau = 0$ . In these coordinates the explicit symmetry of the model is reduced to  $(SO(3) \times U(1))^2$ . Some comments about the relation between our Type IIB Matrix model and the Tiny Graviton Matrix theory of [1] are now in order.

The Tiny Graviton Matrix theory of reference [1] is a  $U(N)$  gauge theory, with two main differences from our proposal. First, the Tiny Graviton Matrix theory is built up with eight non-Abelian scalars plus an additional fixed  $U(N)$  matrix,  $\mathcal{L}_5$ , which is introduced in order to be able to quantize the odd Nambu brackets of the light-cone 3-sphere. This matrix does not have a direct physical interpretation, but it allows to couple the RR 4-form potentials in the action. Therefore, from our point of view we would expect  $\mathcal{L}_5$  to be related to the existence of isometric directions in the background, since in our construction the RR 4-form potentials couple in the action contracted with the Killing vector associated to the isometry. This is in agreement with the discussion in [32]. In this reference it is argued that  $\mathcal{L}_5$  has its origin in M-theory compactified in  $T^2$ . In this compactification one of the directions is the light-cone direction, and the second one is the origin of  $\mathcal{L}_5$  in the

Type IIB theory. This idea becomes explicit in our construction. Indeed, the two isometric directions of the Type IIB action are the direction of propagation of the gravitons and the direction used to construct this action from the action for type IIA gravitons using T-duality. Therefore this action is related to an M-theory action with two isometric directions. One is the direction of propagation of the waves, in this case  $x^-$ , and the other one is the T-duality direction.

Second, the manifest symmetry of the Matrix model in [1] is the full  $(SO(4))^2$  invariance of the background. Our model has the advantage of not depending on the unphysical matrix  $\mathcal{L}_5$  but at the expense of losing the full symmetry of the background.

The differences between the two models become more evident when one looks at their vacuum solutions, as we do in the next subsection. We will see however that both models support fuzzy 3-sphere solutions with the right scaling of the radius with the light-cone momentum in the large  $N$  limit.

### 3.4 The fuzzy sphere solutions

A non-trivial check of the correctness of our Matrix model (3.18) is that it supports fuzzy 3-sphere solutions which agree exactly, in the limit of large number of gravitons, with the classical 3-spheres of [6, 22]. Note that the 3-sphere giant graviton expanding in the spherical part of the geometry [6] and the one expanding in the AdS part [7, 33] of the  $AdS_5 \times S^5$  spacetime are mapped under Penrose limit into the same type of solution, a fact that is reflected in the action through the  $\mathbb{Z}_2$  symmetry  $r \leftrightarrow y$ ,  $X \leftrightarrow Z$ . Therefore we only need to study in detail one of the two solutions.

Let us consider for instance the dual giant graviton solution, i.e. the one expanding into the (Penrose limit of the) AdS part of the geometry.

Our fuzzy 3-sphere ansatz is given by:

$$r = \text{constant}, \quad y = Z^a = 0, \quad a = 1, 2, 3, \quad X^i = \frac{1}{\sqrt{N^2 - 1}} J^i, \quad i = 1, 2, 3, \quad (3.19)$$

where  $J^i$  are  $SU(2)$  generators in an  $N$  dimensional representation (in our conventions  $[J^i, J^j] = 2i\epsilon^{ijk} J^k$ ). That is, we define the fuzzy 3-sphere as an  $S^1$  bundle over a fuzzy 2-sphere. Substituting this ansatz in (3.18) we get

$$S = \frac{N}{R} \int dx^+ \frac{r^2}{2} \left( \mu - \frac{r^2 R}{4\sqrt{N^2 - 1}} \right)^2. \quad (3.20)$$

Since our configuration is static the Hamiltonian is just minus the Lagrangian, and we can compare directly (3.20) with the classical Hamiltonian of [22], which is given in our notation by [10]

$$H = -p_- \frac{r^2}{2} \left( \mu + \frac{2\pi^2 T_3 r^2}{p_-} \right)^2. \quad (3.21)$$

We find that both expressions agree exactly in the large  $N$  limit, once we take into account that  $T_3 = (8\pi^2)^{-1}$ , in units in which  $T_1 = 1$ , and that we are describing the sector of the theory with  $p_- = -N/R$ . The corresponding radii of the giant graviton solutions, given by

$$r^2 = \frac{4\mu\sqrt{N^2 - 1}}{R} \quad (3.22)$$

and

$$r^2 = -\frac{\mu p_-}{2\pi^2 T_3}, \tag{3.23}$$

also agree exactly in this limit. This is a non-trivial check of the validity of our Matrix model (3.18).

Note that in our construction the fuzzy 3-spheres are realized as  $S^1$  bundles over fuzzy 2-spheres. Therefore these vacua have just an explicit  $U(1) \times SO(3)$  symmetry. Although at the classical level the  $SO(4)$  covariance of the 3-sphere is still present in a non-manifest way, the fuzzy 3-sphere consists on an Abelian fibre over a non-Abelian base manifold, and this makes unclear how the whole  $SO(4)$  invariance can be recovered. This set-up is however useful, because the difficulties associated to the fuzzification of odd dimensional spheres (see [34–36]) are avoided by reducing the dimensionality of the fuzzy transverse space from 3 to 2, by means of writing the 3-sphere as an  $S^1$  fibre over an  $S^2$  base manifold, and taking only the  $S^2$  non-commutative. This can be done consistently because the action that we use to describe the gravitons contains an explicit Abelian Killing direction which can be identified with the direction along the fibre<sup>7</sup>.

On the other hand, the fuzzy 3-sphere vacua of the Tiny Graviton Matrix theory of [1] are fully  $SO(4)$ -symmetric<sup>8</sup>. By adding  $\mathcal{L}_5$  to the collection of non-commutative transverse scalars it is possible to construct finite dimensional representations of  $SO(5)$  which can be further reduced to  $N$  dimensional representations of  $SO(4)$ . In this way the fuzzy 3-sphere is constructed from an intermediate fuzzy 4-sphere. Comparing to our description, this construction circumvents the difficulties associated to the fuzzification of the 3-sphere by increasing by one the dimensionality of the fuzzy space, which is precisely the role played by the matrix  $\mathcal{L}_5$ .

Another difference between the fuzzy 3-sphere vacua of [1] and the ones constructed in this paper is that, although the solutions in [1] have the correct scaling of the radius with the momentum in the large  $N$  limit, some non-commutativity still remains (see [34–36]). This is however not the case for the fuzzy 3-sphere solution constructed in this paper, which approaches neatly the classical 3-sphere in the large  $N$  limit, where all the non-commutativity disappears.

## 4. Conclusions

Using the action for coincident Type IIA gravitons constructed in [25, 3]<sup>9</sup> we have reproduced Matrix String theory in the pp-wave background that is obtained by reducing the maximally supersymmetric pp-wave background of M-theory [13, 14]. We have also clarified how in the Matrix model approach the fuzzy 2-sphere solutions of [30], with non-vanishing  $x^4$  position in the transverse space, emerge when D0-brane charge is induced in the configuration.

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<sup>7</sup>In this paper we have identified it with the combination  $(\chi + \tilde{\chi})/2$  in order to have a matrix model which is symmetric under the interchange of the two  $\mathbb{R}^4$  subspaces of the background.

<sup>8</sup>See also [34–36].

<sup>9</sup>And completed in this paper in order to describe the pp-wave background.

In the Type IIB case we have started from the action describing coincident Type IIB gravitons of [23]. This action is connected by duality to the action for non-Abelian D-branes constructed in [27], and has been successfully used in the microscopical description of giant gravitons in  $AdS_5 \times S^5$  [23]. Using this action we have made a proposal for a pp-wave Matrix model which supports fuzzy 3-sphere vacuum solutions with the right behavior in the large  $N$  limit.

Our Matrix model is a one dimensional gauge theory which could be a candidate for the holographic description of strings in the pp-wave background. Indeed,  $x^+$ , which parametrizes the conformal boundary of the pp-wave [37], is its time direction, and the matrix model is compactified along  $x^-$ .

As we have mentioned there is a second candidate for this holographic description<sup>10</sup>, which is the Tiny Graviton Matrix theory of [1]. We have already mentioned some differences between the two constructions. On one hand our Matrix model does not depend on the matrix  $\mathcal{L}_5$ , which lacks a direct physical interpretation, however this happens at the expense of losing the explicit  $SO(4) \times SO(4)$  symmetry of the transverse space, and of the Matrix model in [1]. This is related to the fact that  $\mathcal{L}_5$  is associated to an isometry of the background. Since we have made this isometry explicit in our construction we have reduced the size of the symmetry group.

The existence of these two different Matrix models for the Type IIB pp-wave background could be related to the fact that there is no unique way to quantize diffeomorphisms in a 3-sphere. Therefore one could expect different gauge theories with the right continuum limit. A possible connection between the two Matrix models, that would be interesting to check, is whether our Matrix model can be derived in the approach of [1] by first writing the 3-spheres in the transverse space as  $U(1)$  fibres over  $S^2$  base manifolds, and then quantizing only the Nambu brackets associated to the three transverse scalars building up each 2-sphere, which would now be even dimensional.

Finally, we would like to mention that we have only constructed the bosonic parts of the Type II Matrix models, and that it would be interesting to check their supersymmetry properties. Our starting point, the actions for Type II gravitons, are connected by dualities to (non-Abelian) D-brane actions, for which supersymmetry has been studied (see [42, 43]). Using dualities it should be possible to construct the fermionic parts of these actions. Moreover, since the pp-wave backgrounds are linear perturbations to Minkowski we could use the results in [44]. In the Type IIB case we do not expect however that the invariance under the whole  $PSU(2|2) \times PSU(2|2) \times U(1)$  superalgebra of the pp-wave background [45] will be manifest, due to the explicit breaking  $SO(4) \rightarrow SO(3) \times U(1)$  of our construction.

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<sup>10</sup>See also [38–41].



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